

THE ENERGY BUDGET OF GRBS BASED ON UPDATED PROMPT & AFTERGLOW OBSERVATIONS

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ABSTRACT

We compare the isotropic equivalent 15 – 2000 keV γ -ray energy, E_γ , emitted by a sample of 91 *swift* Gamma-Ray Bursts (GRBs) with known redshifts, with the isotropic equivalent fireball energy, E_{fb} , as estimated within the fireball model framework from X-ray afterglow observations of these bursts. The uncertainty in E_γ , which spans the range of $\sim 10^{51}$ erg to $\sim 10^{53.5}$ erg, is $\approx 25\%$ on average, due mainly to the extrapolation from the BAT detector band to the 15 – 2000 keV band. The uncertainty in E_{fb} is approximately a factor of 2, due mainly to the X-ray measurements' scatter. We find E_γ and E_{fb} to be tightly correlated. The average(std) of $\eta_\gamma^{11\text{hr}} \equiv \log_{10}(E_\gamma/(3\varepsilon_e E_{\text{fb}}^{11\text{hr}}))$ are $-0.34(0.60)$, and the upper limit on the intrinsic spread of η_γ is approximately 0.5 (ε_e is the fraction of shocked plasma energy carried by electrons and $E_{\text{fb}}^{x\text{hr}}$ is inferred from the X-ray flux at x hours). If the uncertainties in the determinations of E_γ and E_{fb} are twice larger than we estimated, then the data imply no intrinsic variance in η_γ . We also find that E_{fb} inferred from X-ray observations at 3 and 11 hours are similar, with an average(std) of $\log_{10}(E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{11\text{hr}})$ of $0.04(0.28)$. The small variance of η_γ implies that burst-to-burst variations in ε_e and in the efficiency of fireball energy conversion to γ -rays are small, and suggests that both are of order unity. The small variance of η_γ and the similarity of $E_{\text{fb}}^{3\text{hr}}$ and $E_{\text{fb}}^{11\text{hr}}$ further imply that ε_e does not vary significantly with shock Lorentz factor, and that for most bursts the modification of fireball energy during the afterglow phase, by processes such as radiative losses or extended duration energy injection, are not significant. Finally, our results imply that if fireballs are indeed jets, then the jet opening angle satisfies $\theta \geq 0.1$ for most cases. Extending our analysis to late times we find a significant reduction in E_{fb} , $\langle \bar{E}_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{2\text{d}} \rangle = 1.4$, consistent with jet breaks on a 1 d time scale in a significant fraction of the bursts. These results are consistent with the main results of Freedman & Waxman (2001), which were based on a much smaller sample of GRBs.

Subject headings: cosmology: observations– γ -ray: sources– γ -ray: bursts

1. INTRODUCTION

GRBs are the most powerful explosions in the Universe, and include the highest redshift objects observed. The widely accepted phenomenological interpretation of these cosmological sources is the so called "Fireball (FB) model" (Paczynski 1986; Goodman 1986). In "optically thin" versions of this model, the energy carried by the hadrons in a relativistic expanding wind (fireball) is dissipated through internal shocks between different parts of plasma. These shocks reconvert a substantial part of the kinetic energy to internal energy, which is then radiated as γ -rays by synchrotron and inverse-Compton radiation of shock-accelerated electrons (Rees & Mészáros 1992; Narayan, Paczyński & Piran 1992). Alternatively, photospheric models for the prompt emission have been proposed (e.g. Pe'er et al. 2012; Beloborodov 2013) as being significant contributors to shaping the prompt emission. Regardless of the mechanism responsible for the emission of the prompt gamma-rays, the fireball is expected to drive a shockwave into its surrounding medium, which decelerates as it encompasses an increasing amount of mass and is believed to be responsible

for the afterglow emission on time scales of minutes to years (Paczynski & Rhoads 1993; Meszaros & Rees 1997; Waxman 1997; Sari et al. 1998). Long term afterglow observations led to the conclusion (Rhoads 1999; Sari et al. 1999; Racusin et al. 2009) that the fireball is not spherical but rather jet-like.

The model for the afterglow phase is completely defined by the fireball energy, E_{fb} , the jet opening angle θ , the fraction of shocked plasma energy carried by electrons and magnetic field, ε_e and ε_B , the spectral index of the power-law energy distribution of shock accelerated electrons, p , and the density of the plasma into which the shock expands, n . While afterglow observations are in general in good agreement with the model, it remains difficult to tightly constrain the parameters of the model based on observations. This is due to the fact that a complete calibration of the parameters requires a determination of the three breaks in the spectrum (cooling, peak and self absorption frequencies), which in turn requires a multi wavelength coverage of the afterglow over hundreds of days. In particular, the fireball energy, and hence the efficiency with which this energy is converted to γ -rays, can be determined in only a few cases, for which adequate spectral coverage is available (e.g. GRB970503, Frail et al. 2000).

Freedman & Waxman (2001, hereafter FW01) have shown that it is possible to estimate the fireball energy (note that throughout this work, 'fireball energy', as well as GRB energy, refer to the isotropic equivalent energies) carried by electrons ($\varepsilon_e E_{\text{fb}}$) using a measurement at a

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single time of the flux at a frequency for which the emission is dominated by fast cooling electrons (i.e. electrons for which the cooling time is shorter than the dynamical expansion time), and that this estimation is weakly dependent on poorly constrained model parameters. The physical basis underlying this estimate is that the luminosity at such a frequency corresponds to the rate at which energy is deposited in shock accelerated electrons, $\sim \varepsilon_e E_{\text{fb}}/t$. Since such observations have been carried out for many GRBs, this allows one to infer the efficiency of γ -ray production for a large sample of GRBs. Using this method, it was found in FW01 that E_γ and $\varepsilon_e E_{\text{fb}}$, inferred from X-ray observations at ~ 11 hours, are strongly correlated, implying that the burst-to-burst variations in ε_e and in the efficiency of fireball energy conversion to γ -rays are small, and suggesting that both are of order unity. It was also pointed out that if GRB fireballs are indeed jet-like, then the jet opening angle should satisfy $\theta \geq 0.1$ for most bursts (for smaller opening angles, sideways expansion of the jet would occur at earlier times, thus significantly affecting the inferred isotropic equivalent energy, at $t < 11$ hours). While these results are significant, they were based on a small sample of GRBs and their afterglows: a total of 13 GRBs of which only 7 had measured redshifts.

The applicability of the results and conclusions of FW01 to the majority of GRBs is challenged by more recent studies, which suggest that the fireball energy is significantly modified on a time scale of hours following the burst by extended energy injection or significant radiative losses. Nousek et al. (2006) suggested, based on an analysis of 9 lightcurves from a sample of 27 GRBs, that the fireball energy is increased by a factor ≥ 4 due to energy injection on a timescale mostly up to 3 hours (with two exception of up to 5 and 11 hours); Zhang et al. (2006) argue that a good fraction of GRBs (though they do not quantify how many) have shallow decay phases on timescales of up to a few hours, and interpret them as corresponding to continual energy injection, which increases the fireball energy by a factor of up to 10, and Panaitescu & Vestrand (2012) find significant energy injection to be ubiquitous in afterglows through analyzing the consistency of breaks with the fireball model. Analysing two GRB afterglows Berger et al. (2004) concluded, based on Yost et al. (2003), that 50% and 90% of the fireball energy of the two bursts was radiated away during the early (3 hours and 6 days) afterglow. It was furthermore argued (Sari & Esin 2001) that the X-ray flux based estimate of the fireball energy E_{fb} should be strongly affected by inverse-Compton (IC) energy loss of the electrons, which may lead to a significant fraction of the electron energy being radiated at (unobserved) frequencies well above the X-ray band (where the emission is dominated by synchrotron radiation), and which is expected to strongly vary between bursts.

Such significant modifications of the fireball energy (or of the estimated fireball energy, e.g. due to IC suppression of the synchrotron emission), which vary from burst to burst and are not expected to be correlated with the prompt γ -ray emission, would introduce a large scatter to the E_γ/E_{fb} ratio, which would be inconsistent with the results of FW01. In this paper we expand the analysis of FW01 to a large sample of Swift GRBs, in order to examine whether the tight correlation of E_γ and E_{fb} ,

and the implications of such a tight correlation, hold for the majority of the GRBs. The large sample and the improved afterglow data available enable us to quantify the uncertainties more reliably and thus also to draw quantitative conclusions.

The GRB sample used in our analysis is described in § 2. The methods used for estimating E_γ and E_{fb} , and the uncertainties in these estimates, are described in § 3 and in § 4 respectively. The correlation between E_γ and E_{fb} is analyzed in § 5. The implications of the relatively tight correlation between E_γ and E_{fb} are discussed in § 6, and our conclusions are summarized in § 7.

2. THE GRB SAMPLE

Our sample consists of 91 long GRBs with known redshifts, which were detected by Swift in the period between May 2006 and August 2009 (from GRB060502A to GRB090812). The total number of long GRBs that were detected in this period, and for which redshifts are known, is 112 (7 short, $T_{90} < 1.8$ s, GRBs with known redshift that were detected during this period are not included in our sample). 21 of the 112 GRBs were excluded from the analysis due to the following reasons: for 1 GRB the required BAT data are not available; 14 GRBs do not have satisfactory XRT data at the 3-11 hours time interval (For 2 of these bursts no XRT data are available, for 8 bursts no data are available at $t > 4$ hr, and for 4 there are only 3 data points in the time range of $3\text{hr} < t < 11$ hr. We note that the lack of data for these bursts is unlikely to be due to low X-ray fluxes, and is likely due to low sampling cadence, since the existing data points show relatively high fluxes.); 6 bursts have late time (close to 3 hours) flares or clearly rising lightcurves around the relevant times.

The exclusion of the latter group of 6 GRBs from our analysis implies that our conclusions may not apply to a small minority, $\sim 5\%$, of the long GRB population.

3. ESTIMATING E_{FB}

The fireball energy carried by electrons can be estimated from a measurement of the flux density f_ν at a frequency ν , which is above the cooling frequency (corresponding to the frequency of radiation emitted by electrons with cooling time equal to the dynamical time), using equations 4 and 5 of FW01:

$$\varepsilon_e E_{\text{fb}} = (C_2 C_3)^{-1/2} C_1^{-1} \frac{d_L^2}{1+z} \nu t f_\nu(\nu, t) Y^\varepsilon, \quad (1)$$

where

$$Y \equiv C_1 C_3^{1/2} C_2^{-3/2} \varepsilon_e^{-3} \varepsilon_B^{-1} d_L^{-2} \nu t^2 f_\nu^{-1}(\nu, t), \quad \varepsilon \equiv \frac{p-2}{p+2} \quad (2)$$

and C_1, C_2, C_3 are numerical constants taken from FW01, $C_1 = 1.4 \times 10^{-21} \text{cm}^3/2$, $C_2 = 6.1 \times 10^{-5} \text{s}^{3/2} \text{g}^{-1/2} \text{cm}^{-1}$ (for $p = 2.2$) and $C_3 = 6.9 \times 10^{39} \text{s}^{-3/2} \text{g}^{-1/2} \text{cm}^{-2}$. The inferred energy is independent of the density of the plasma into which the fireball expands, and very weakly dependent on the value of ε_B for $p \approx 2$. For our nominal energy estimates we use $\varepsilon_e = \varepsilon_B = 1/3$. The dependence on ε_e is shown explicitly in all our results.

For the spectral index, we adopt in our analysis a value of $p = 2.2$. We show below that such a universal value

is supported by both the afterglow spectra (§ 3.1) and the time dependence of afterglow X-ray flux (§ 3.2). As discussed in some detail in § 6.3, the tight correlation between E_{fb} and E_{γ} further supports a universal value of p . Using a value of $p = 2.4$ instead of $p = 2.2$ increases the estimate of E_{fb} by 60% (FW01).

Eqs. (1-2) yield an accurate estimate of $\varepsilon_e E_{\text{fb}}$ under the assumptions that the emitting plasma is well described by the self-similar spherical fireball model and that electron acceleration is well characterized by time independent ε_e and p . When these assumptions hold, the X-ray flux should drop with time as $f_X \propto t^{(2-3p)/4}$, and E_{fb} inferred using eq. (1) should be independent of t . A measure of the accuracy of the determination of $\varepsilon_e E_{\text{fb}}$, and a test of the validity of the above assumptions, is therefore obtained by comparing the values of $\varepsilon_e E_{\text{fb}}$ inferred from the X-ray flux at significantly different times. When the assumptions described above are not valid, eqs. (1-2) provide a less accurate estimate of $\varepsilon_e E_{\text{fb}}$.

We show in § 3.2 that the values of E_{fb} inferred using eq. (1) at 3 hr and 11 hr are similar (implying $d \log f_X / d \log t \simeq -1$): we find that $< \log_{10}(E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{11\text{hr}}) >= 0.04$ and that only $\approx 20\%$ of the bursts have $E_{\text{fb}}^{3\text{hr}}$ and $E_{\text{fb}}^{11\text{hr}}$ values differing by more than a factor of 2. These results support the validity of the model assumptions described above. They may appear to be contradictory to the claims that “flat” X-ray lightcurves (with $d \log f_X / d \log t$ significantly larger than -1), and hence significant energy injection, are common (e.g. Nousek et al. 2006). This issue is discussed in § 3.3.

Finally, we note that the uncertainty in the energy estimate should include a systematic uncertainty due to the uncertainty in the values of the constants $\{C_1, C_2, C_3\}$, which vary by factors of a few between different models of the afterglow synchrotron emission (see for example the discussion in Granot, Konigl & Piran 2006). We do not include in our analysis a treatment of this systematic uncertainty, since a modification of $\{C_1, C_2, C_3\}$ will modify the values of E_{fb} and of E_{γ}/E_{fb} by a fixed factor for all bursts, but will not affect the fractional scatter of these values. It will thus not affect the main conclusions of this work.

3.1. Constraints on p from afterglow spectra

Following FW01, we present below evidence for the validity of the approximation of $p \approx 2$ based on the spectral index of the afterglow emission, determined from the X-ray and optical fluxes, $\beta_{ox} \equiv -\ln(f_x/f_o)/\ln(\nu_x/\nu_o)$. At times when the cooling frequency is below the X-ray band, we expect $(p-1)/2 < \beta_{ox} < p/2$ (with the lower limit obtained when the cooling frequency is in the X-ray band, and the upper limit when it is in the optical band).

In fig. 1 we show β_{ox} at 11 hours for the first 38 GRBs in the sample (up to GRB071031), as given by Zheng et al. (2009). For 75% of the bursts, β_{ox} is in the range of 0.6-1.1, which is the range expected for $p = 2.2$. Of these bursts for which the measured spectral index does not lie in this range, one has $\beta_{ox} > 1.1$ while the others have $\beta_{ox} < 0.6$. Such values are in principle not consistent with the shape of the spectrum in the fireball model for $p = 2.2$ at times when the cooling frequency lies below the measured X-ray band. However, as already noted by FW01, dust extinction in the host

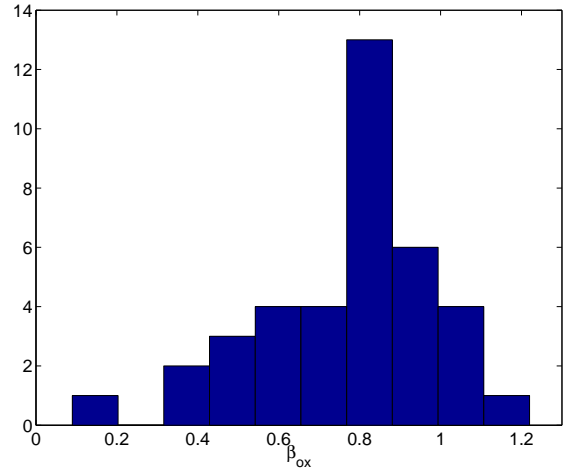


FIG. 1.— A histogram of the effective spectral index β_{ox} for a subset of 38 bursts within our sample, given by Zheng et al. (2009).

galaxy, which suppresses the optical flux, can reduce the observed value of β_{ox} . Zheng et al. (2009) find that low values of β_{ox} typically correspond to high values of hydrogen column density, which serves as a proxy for the strength of local dust extinction. Moreover, they state (referring to previous work) that for several of these low β_{ox} bursts, detailed studies of the optical afterglow allowed one to measure the optical extinction values, thus raising the value of β_{ox} . Finally, we have compared the group of bursts with low β_{ox} in the sample to the whole sample and found both groups to have the same average E_{fb} and E_{γ}/E_{fb} , thus indicating that the low β_{ox} bursts do not represent a separate population.

3.2. f_X and E_{fb} at 11 and 3 hrs

For each GRB we have estimated the 10 keV flux density at 11 and 3 hours after the GRB trigger by fitting a power-law temporal behavior to the Swift X-ray lightcurve between 3×10^3 s and 6×10^4 s. Due to the rapid decay and to the flares often characterizing the X-ray light curve at early time, we do not use $t < 3 \times 10^3$ s data (except for 4 GRBs, which clearly show a regular behavior of the lightcurves at earlier times and for which the earlier time data are necessary in order to obtain a reliable interpolation at 3 hours). Due to the possible appearance of a jet-break suppression of the flux at late time, $t > 1$ d, we do not use $t > 6 \times 10^4$ s data (except for 12 GRBs, for which a late break in the lightcurve is clearly absent and for which later data are necessary to obtain a reliable interpolation at 11 hours).

The uncertainties in the derived X-ray fluxes at 3 and 11 hours are dominated by the scatter in the reported X-ray flux measurements, as illustrated in figure 2. We use this scatter as indicative of the flux uncertainty, rather than the reported flux uncertainties, since the latter do not appear to be consistent with the scatter of the data points. The fluxes at 3 and 11 hours are estimated using a least square fitting of a first degree polynomial in the $\log t - \log f_X$ plane, and their uncertainties are determined such that 90% of the data points lie within the uncertainty. We note that for 20% of the bursts, for which the last measured X-ray data point is at a time earlier than 5 hours after the trigger, the extrapolation

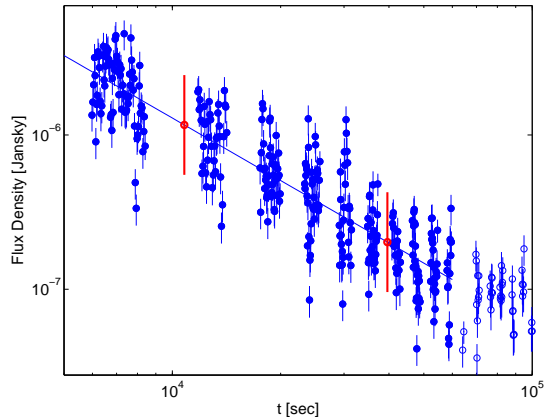


FIG. 2.— XRT flux measurements for GRB090618. The straight line is the power-law time decay fit for the range of interest, based on which the fluxes at 3 and at 11 hours are inferred (red circles). The red lines represent the estimated flux uncertainties (see text). The empty circles represent data points at $t > 6 \times 10^4$ s, which are not taken into account in determining the fluxes at 3 and 11 hours.

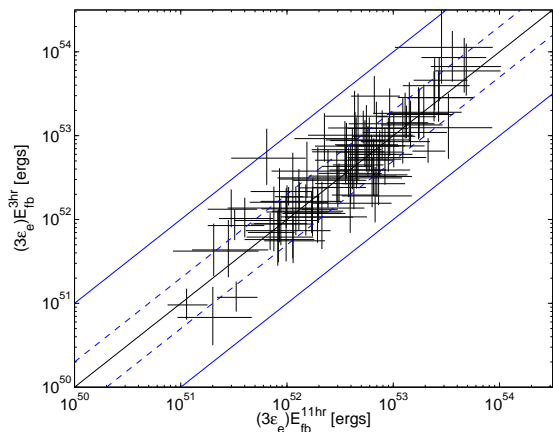


FIG. 3.— The relation between E_{fb} inferred from the X-ray fluxes at 3 and 11 hrs. The dashed (solid) blue lines correspond to the energy estimates at 3 and 11 hrs being within a factor of 2 (10) of each other.

of the power law fit to 11 hours makes a significant contribution to the uncertainty.

The average(std) of $d \log f_X / d \log t$ we obtain from our fits to the X-ray light curves between 3×10^3 s and 6×10^4 s are -1.25 (0.52), consistent with the value of -1.1 expected for $p = 2.2$. The deviation of the decay rate from this value is larger than the estimated error in the determination of the decay rate for $< 20\%$ of the bursts. This result is illustrated in figure 3, showing the tight correlation between the fireball energies estimated at 3 and 11 hours. The average(std) of $\log_{10}(E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{11\text{hr}})$ are 0.04 (0.28). Given our estimated uncertainties in inferring E_{fb} , the data are consistent with no intrinsic variance in the ratio $E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{11\text{hr}}$. Out of the 91 bursts in the sample, 11(14) have a fireball energy estimated at 3 hours which is less than half (more than twice) that estimated at 11 hours. We show in § 5 that the average of η_γ is similar for bursts with small (less than a factor of 2) and large (more than a factor of 2) differences between $E_{\text{fb}}^{3\text{hr}}$ and $E_{\text{fb}}^{11\text{hr}}$, while the variance of η_γ is somewhat larger for the latter group.

3.3. Flat light curves

The results reported in the preceding paragraph may appear to be contradictory to the claims that “flat”, i.e. $d \log f_X / d \log t$ significantly larger than -1 , X-ray lightcurves (and hence significant energy injection) are common (e.g. Nousek et al. 2006). This is, however, not (necessarily) the case. Nousek et al. (2006) concluded that the canonical lightcurve contains a shallow decay phase based on an analysis of 27 bursts out of which only 9 showed flat decay phases, and only 5 ($< 20\%$) showed such behavior extending to $t > 6 \times 10^3$ s. Our analysis, based on a much larger and complete sample, shows that the fraction of afterglows showing a flat phase leading to a factor 2 or larger increase in the inferred E_{fb} between 3 and 11 hr is $\lesssim 10\%$.

At times earlier than 3×10^3 s, the lightcurves are in general less regular, as was discussed in § 3.2, with many of the lightcurves showing both steeper and flatter time dependence compared to later times. In particular, roughly 25% of the bursts show lightcurves which are significantly flatter at early times compared to late times. However, as we explain in detail in § 6.6, the contribution of possible early energy injection to the total fireball energy cannot in general be significant.

4. ESTIMATING E_γ

The γ -ray spectrum of GRBs is typically described using a Band function (Band et al. 1993),

$$\frac{dN}{dE} = \begin{cases} A \left(\frac{E}{50 \text{ keV}} \right)^\alpha e^{-\frac{E(2+\alpha)}{E_{\text{peak}}}} & \text{if } E < E_{\text{break}} \\ A \left(\frac{E_{\text{break}}}{50 \text{ keV}} \right)^{(\alpha-\beta)} \left(\frac{E}{50 \text{ keV}} \right)^\beta & \text{if } E \geq E_{\text{break}} \end{cases} \quad (3)$$

where $E_{\text{break}} \equiv \frac{(\alpha-\beta)E_{\text{peak}}}{(2+\alpha)}$. Due to the limited energy range of the BAT detector, 15 to 150 keV (Barthelmy et al. 2005), the typical values inferred for the Band function parameters based on BAT data differ from the typical values inferred based on data from BATSE, which is sensitive in the range of 30 to 2000 keV (Kaneko et al. 2006). While the average low energy index α in the BAT sample used here, -1.11 , is similar to the average value of the BATSE GRBs, -1.08 (Kaneko et al. 2006), the peak energy E_{peak} and the high energy index β inferred from the BAT and from the BATSE data are quite different. In particular, the average peak energy obtained for BATSE spectra is 260 keV (Kaneko et al. 2006), outside the BAT energy band. Thus, in order to determine E_γ in the energy range of 15 to 2000 keV for the BAT GRBs, we use a Band function with α and flux normalization as given in the Swift data archive (determined by the BAT spectrum), but use the average values of the BATSE catalog, $E_{\text{peak}} = 260$ keV and $\beta = -2.4$ (Kaneko et al. 2006). This procedure introduces of course an uncertainty in the estimated value of E_γ , due to the uncertainty in the extrapolation of the spectrum up to 2000 keV. We estimate the uncertainties using extrapolations to high energy with the extreme values obtained in the BATSE catalog for E_{peak} and β : 150 keV and 500 keV for E_{peak} and -2.6 and -2.0 for β .

The method for estimating E_γ and its uncertainty depends on the values of the Band function parameters given in the Swift data archive, as explained in detail below. We note that in all cases, the low end of the

model spectrum we adopt is identical to that found by BAT (and its flux integrated over the 15-150 keV range matches the BAT reported integrated flux).

- For β in the range of $[-4, -2]$, which holds for $\approx 15\%$ of the bursts in our sample, the flux per logarithmic photon energy interval, $E^2 dN/dE$, is decreasing with E . We therefore estimate the [15,2000] keV fluence by integrating the BAT Band function up to 2000 keV, and the uncertainty range by extrapolating the BAT spectrum beyond 150 keV using a $\beta = -2.6$ and a $\beta = -2.0$ power-laws.
- For $\beta > -2$, which holds for $\approx 55\%$ of the bursts, $E^2 dN/dE$ is increasing with E . We therefore estimate the [15,2000] keV fluence by extrapolating the BAT Band function to 260 keV, followed by a $\beta = -2.4$ power-law at higher energy. The lowest value of energy uncertainty range is obtained by extrapolating the BAT spectrum beyond 150 keV using a $\beta = -2.6$ power-law, and the highest value by extrapolating the BAT Band function to 500 keV followed by a $\beta = -2.0$ power-law at higher energy.
- For $\beta < -4$, which holds for $\approx 30\%$ of the bursts, E_{break} inferred by the SWIFT analysis is well above the BAT band. We therefore estimate the [15,2000] keV fluence by extrapolating the BAT Band function to 260 keV, followed by a $\beta = -2.4$ power-law at higher energy. The lowest value of the energy uncertainty range is obtained by extrapolating the BAT Band function to 500 keV followed by a $\beta = -2.6$ power-law, and the highest value by extrapolating the BAT spectrum beyond 150 keV using a $\beta = -2.0$ power-law.

The ratios of the [15,2000] keV fluences, inferred as described above, and the BAT [15 150] keV fluences are in the range of 1.02 to 4.26, with an average of 2.25. The uncertainties due to the extrapolation are $\approx 25\%$ on average, reaching a factor of 2 for some bursts. In addition to the uncertainty introduced by the extrapolation to high energies, we include in our analysis the uncertainty in the BAT fluence as reported in the SWIFT archive. The average value of the latter uncertainty is 7%, and it amounts to no more than 26% for any burst in the sample.

5. THE CORRELATION BETWEEN E_γ AND E_{FB}

Using the methods described in the preceding subsections we find the average(std) of $\log_{10}(E_\gamma/1\text{erg})$ and of $\log_{10}(3\varepsilon_e E_{\text{fb}}^{11\text{hr}}/1\text{erg})$ to be 52.3(0.7) and 52.5(0.6) respectively, very close to the values inferred in FW01. As can be seen in figure 4, $\log(E_\gamma)$ and $\log(E_{\text{fb}})$ are linearly correlated, with a high correlation coefficient of ≈ 0.6 . For the average(std) of $\eta_\gamma^{11\text{hr}} \equiv \log_{10}(E_\gamma/(3\varepsilon_e E_{\text{fb}}^{11\text{hr}}))$ we find $-0.34(0.60)$, consistent with the results of FW01, 0.01(0.5) (and also with those of D’Avanzo et al. 2012, who give their nominal results with a normalization of $\varepsilon_e = 0.1$).

Given the estimated uncertainties in the derived values of E_γ and of $\varepsilon_e E_{\text{fb}}^{11\text{hr}}$, the hypothesis that their ratio is universal (i.e. the hypothesis of no intrinsic variance in

TABLE 1
Average values and Standard deviations

time set ^a	3 hr all	3 hr tight	11 hr all	11 hr tight
sample size	91	66	91	66
$\langle \log_{10}((3\varepsilon_e)E_{\text{fb}}) \rangle$	52.6	52.6	52.5	52.6
$\text{var}(\log_{10}((3\varepsilon_e)E_{\text{fb}}))$	0.6	0.6	0.6	0.5
$\langle \eta_\gamma \rangle$	-0.36	-0.43	-0.34	-0.41
$\text{var}(\eta_\gamma)$	0.55	0.51	0.60	0.51
χ^2 pdf ^b	3.3	2.9	3.6	2.6
intrinsic variance ^c	0.41	0.36	0.50	0.36

^a“all” is the entire burst sample analyzed in this work, “tight” is a subset for which the fireball energy estimates at 3 hours and at 11 hours are within a factor of 2 of each other.

^b χ^2 , as defined in the text.

^crequired to obtain $\chi^2 = 1$.

$\eta_\gamma^{11\text{hr}}$) is inconsistent with the data (resulting in χ^2 per degree of freedom of ≈ 4). In order to estimate the intrinsic variance of $\eta_\gamma^{11\text{hr}}$, we assume that it follows a gaussian distribution with a variance s . s is determined by equating χ^2 to 1 in the equation $\chi^2 = (N - 1)^{-1} \sum \frac{(y_i - x_i - \eta_\gamma)^2}{\sigma_i^2 + s^2}$, where x_i, y_i are $\log(E_\gamma), \log(E_{\text{fb}})$, and σ_i represents the uncertainty in the energy estimates ($\sigma_i^2 = \sigma_{x_i}^2 + \sigma_{y_i}^2$) (minimizing χ^2 while setting $s = 0$ gives the nominal value of η_γ , which is then used in computing s). We find the intrinsic variance to be approximately 0.5. We note, though, that this value is sensitive to the uncertainties in E_γ and E_{fb} . For example, if these uncertainties are twice larger than we estimated, then the data implies no intrinsic variance in η_γ .

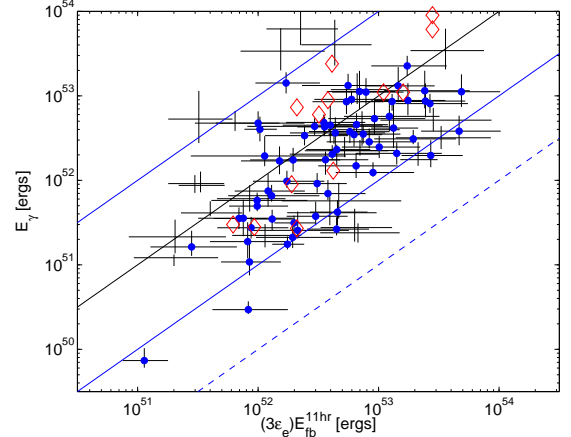


FIG. 4.— The relation between E_γ and $E_{\text{fb}}^{11\text{hr}}$. The GRBs marked with filled circles are those for which the fireball energy estimates at 3 hours and 11 hours lie within a factor 2 of each other. We show for comparison the results obtained by FW01 (red diamonds). Solid lines correspond to $E_\gamma/(3\varepsilon_e)E_{\text{fb}}^{11\text{hr}} = 0.1, 1, 10$ and the dashed line to $E_\gamma/(3\varepsilon_e)E_{\text{fb}}^{11\text{hr}} = 0.01$.

Repeating the above analysis using X-ray observations at 3 hours, we find the average(std) of $\log_{10}(3\varepsilon_e E_{\text{fb}}^{3\text{hr}}/1\text{erg})$ and $\eta_\gamma^{3\text{hr}} \equiv \log_{10}(E_\gamma/3\varepsilon_e E_{\text{fb}}^{3\text{hr}})$ to be 52.6(0.6) and $-0.36(0.55)$ respectively, similar to the values estimated at 11 hours.

Table 1 gives the values of the average and variance of the fireball energy and of η_γ , obtained with and without bursts for which the fireball energy estimates at 3 and 11

hours differ by more than a factor of 2. As can be seen from the table, the results obtained with and without these bursts are very similar. We note that, as shown in Fig. 6, the variance of η_γ is larger for bursts characterized by larger differences between $E_{\text{fb}}^{3\text{hr}}$ and $E_{\text{fb}}^{11\text{hr}}$ (while its average value is nearly independent of this difference, Fig. 7).

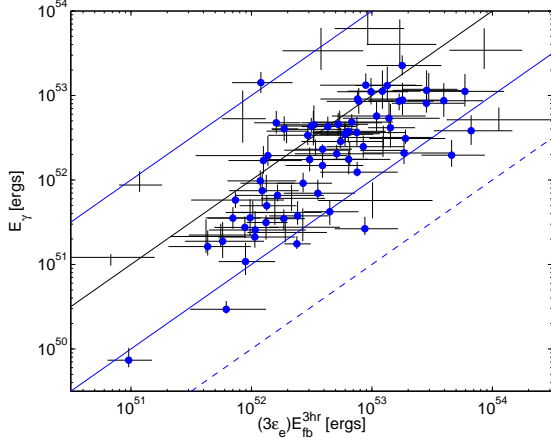


FIG. 5.— Same as fig. 4, for E_{fb} inferred from the X-ray flux at 3 hrs (instead of at 11 hrs).

While η_γ lies within the range of ~ 0.1 to ~ 1 for most of the GRBs in our sample, there are 5 GRBs (GRB080607, GRB080319B, GRB061110B, GRB061007, GRB060510B) with exceptionally large values of η_γ , $\eta_\gamma(3\epsilon_e)^{-1} > 8$. Out of these, 4 bursts have exceptionally steep lightcurves between 3 and 11 hours relative to what is expected from the fireball model. One possible explanation for such behavior could be that these GRBs suffer from significant radiative losses at these times, implying that their E_γ/E_{fb} is actually lower than inferred by using the estimates of $E_{\text{fb}}^{3\text{hr}}$ and $E_{\text{fb}}^{11\text{hr}}$ (Estimating the radiative losses for particular bursts must be done using extensive lightcurve fitting to constrain the fireball model parameters, which is beyond the scope of this work). It should however be pointed out that, as shown in Fig. 8, for these bursts the ratio of our inferred [15,2000] keV fluence to the [15,150] keV measured BAT fluence is 4, the largest in our sample. This implies that the E_γ and η_γ estimates of these bursts are more sensitive to systematic errors in our extrapolation method.

6. DISCUSSION

As mentioned in the introduction, significant modifications of the fireball energy, or of the estimated fireball energy, which vary from burst to burst and are not expected to be correlated with the prompt γ -ray emission, would introduce a large scatter to the E_γ/E_{fb} ratio. Thus, the relatively tight correlation we find (§ 5) between E_γ and E_{fb} sets relatively tight constraints on the combined effect of processes that lead to such modifications. Quantitatively, the intrinsic variance in η_γ , which is inferred from the scatter in the $E_\gamma - E_{\text{fb}}$ correlation, sets an upper limit of a factor of 2-3 on the scatter of the E_γ/E_{fb} ratio due to the combined effects of these processes. In what follows we discuss the main implications of the tight correlations we find between E_γ and E_{fb} and

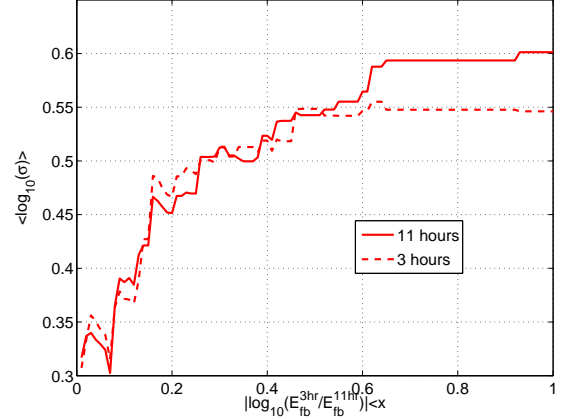


FIG. 6.— The standard deviation of η_γ of that part of the bursts within the sample for which the fireball energy estimated at 3 and 11 hours is within a factor of 10^x .

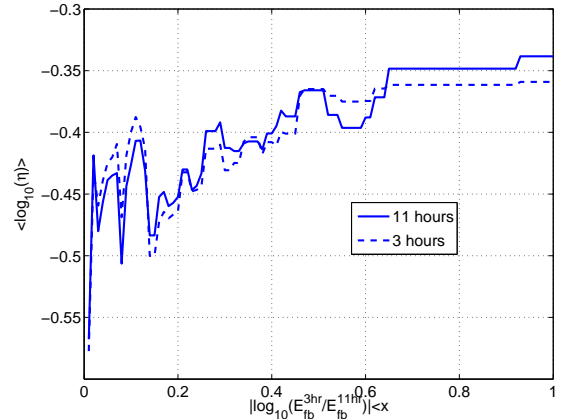


FIG. 7.— The average value of η_γ of that part of the bursts within the sample for which the fireball energy estimated at 3 hrs and at 11 hrs are within a factor of 10^x of each other.

between $E_{\text{fb}}^{11\text{hr}}$ and $E_{\text{fb}}^{3\text{hr}}$.

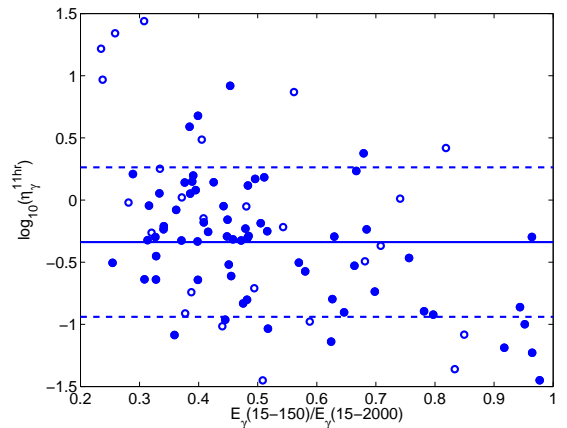


FIG. 8.— η_γ versus the fraction of the total estimated GRB energy which is contained in the BAT range. Empty circles represent bursts for which $E_{\text{fb}}^{11\text{hr}}$ and $E_{\text{fb}}^{3\text{hr}}$ differ by more than a factor of 2. The solid line shows the average of η_γ , the dashed lines are separated from the average by the variance of η_γ .

6.1. Jet opening angle and "off-axis" detections

The fact that the fireball energy estimates at 3 and 11 hours are similar indicates that in general jet breaks do not occur well before 11 hours, since that would cause the inferred $E_{\text{fb}}^{11\text{hr}}$ to be on average significantly lower than $E_{\text{fb}}^{3\text{hr}}$. Since the relation between the jet opening angle θ_j and the jet break time t_j is estimated as (e.g. Livio & Waxman 2000)

$$\theta_j \approx 0.12 (E_{\text{iso},53}/n_0)^{-1/8} t_{j,\text{day}}^{3/8}, \quad (4)$$

we conclude that typical opening angles are greater than 0.1, as inferred in a similar manner by FW01.

A similar argument also implies that the population of observed GRBs can not be dominated by jets observed initially (during γ -ray emission) "off-axis", i.e. with an angle θ between the line of sight and the jet axis, which is larger than θ_j . In such cases, the evolution of the fireball and of the emission of the afterglow radiation are not described by the simple self-similar model leading to eqs. (1-2), which are used to infer E_{fb} , the luminosity of the prompt γ -ray emission would vary strongly depending on θ (e.g. Lundman Pe'er & Ryde 2013), and the tight correlation between E_γ and E_{fb} would be a coincidence.

In order to test for the possible presence of jet breaks at $t > 11$ hr, and to demonstrate the sensitivity of our analysis method to such breaks, we have derived $E_{\text{fb}}^{2\text{d}}$ for the 73 bursts within the sample for which X-ray light curves extending to $t > 2$ d (with at least 2 measurement points at $t > 1$ d) are available, using data points at $t > 1$ d only. For these bursts we find $\langle \log_{10}(E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{2\text{d}}) \rangle = 0.16$, significantly larger than the uncertainty in estimating $\langle \log_{10} E_{\text{fb}} \rangle$, which is ≈ 0.07 . The hypothesis that the average energies at 3 hr and 2 d are equal is rejected at a $\approx 85\%$ confidence level based on a standard two sample t-test (Recall that in § 3 we have found that $\langle \log_{10}(E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{2\text{d}}) \rangle = 0.04$, consistent with no change in $\langle \log_{10} E_{\text{fb}} \rangle$ between 3 hr and 11 hr). A possible explanation of the $\approx 40\%$ reduction in $\langle E_{\text{fb}} \rangle$ between 3 hr and 2 d is a jet break at $t < 2$ d. A precise characterization of the nature of jet breaks is beyond the scope of this work, but the $\approx 40\%$ drop could correspond (in two extreme cases) to a universal jet break time of ≈ 30 hr or to $\approx 1/2$ the bursts having a jet break at 11 hr.

6.2. Variations of the γ -ray emission on small angular scales

Due to the relativistic expansion of the fireball, the flux we observe is obtained from a conical section of the fireball (around the line of sight) with an opening angle of $1/\Gamma$, where Γ is the (time dependent) expansion Lorentz factor. The GRB γ -rays are expected to be emitted at the highly relativistic, $\Gamma = \Gamma_\gamma \sim 10^{2.5}$, phase of fireball expansion, while the X-rays observed at ~ 10 hrs are expected to be emitted after deceleration to $\Gamma = \Gamma_X \sim 10^1$. Thus, the isotropic equivalent fireball energy, E_{fb} , is determined by an average of the jet properties over angles $\sim 1/\Gamma_X \sim 0.1$, much larger than those from which γ -ray emission is observed, $\sim 1/\Gamma_\gamma < 0.01$. If the emission of γ -rays were to vary significantly over angular scales $0.01 \sim 1/\Gamma_\gamma < \Delta\theta < 1/\Gamma_X \sim 0.1$, a large scatter would be introduced to the E_γ/E_{fb} ratio. The absence of such large scatter implies that the jet's γ -ray emission does

not vary significantly over $0.01 < \Delta\theta < 0.1$ angular scales within the jet opening angle θ_j .

6.3. Variability in the microscopic parameters of the afterglow model: ε_e , p

Significant burst to burst variations in ε_e and/or p would lead to a significant scatter in the E_γ/E_{fb} ratio, while significant dependence of ε_e and/or p on the shock Lorentz factor would lead to a significant deviation from unity of the ratio $E_{\text{fb}}^{11\text{hr}}/E_{\text{fb}}^{3\text{hr}}$. Our results therefore indicate that ε_e and p are uniform between bursts, which is reasonable given the fact that they are determined by the micro-physics, and that their values do not strongly depend on the shock Lorentz factor.

We note that some earlier studies have found significant scatter in ε_e . For example, Yost et al. (2003) find values in the range of $0.12 - 0.34$ by fitting multiband observations to 4 different afterglows, and Berger et al. (2004) find a factor of 4 difference between the two afterglows for which they present their analysis. While the intrinsic variance we find does not exclude such scatter in ε_e , it would imply that almost all the intrinsic variance we find in the E_γ/E_{fb} ratio is due to the variance in ε_e . This, in turn, would imply that the variance due to burst-to-burst variations in γ -ray production efficiency is negligible. Since ε_e is a parameter determined by microscopic processes, while the γ -ray production efficiency depends on the macroscopic properties of the flow (e.g. Lorentz factor variability within the fireball wind, see § 6.4), this appears to be unlikely.

6.4. Variability in the GRB γ -ray production efficiency

Strong burst-to-burst variations in the efficiency of the conversion of the fireball energy to γ -rays would lead to a large scatter in the E_γ/E_{fb} ratio. The absence of such scatter implies that the efficiency is not highly variable between bursts. This is a strong constraint on the fireball model, since significant variations are natural to expect in most of its variants. For example, in models where the emission of γ -rays follows from the conversion of fireball kinetic energy to internal energy by internal collisions within the wind, the efficiency strongly depends on the amplitude and structure of Lorentz factor variations within the expanding fireball wind. Uniform efficiency suggests an efficient process (with order unity efficiency) of converting fireball energy to radiation.

6.5. Radiative losses

The tight correlation between $E_{\text{fb}}^{11\text{hr}}$ and $E_{\text{fb}}^{3\text{hr}}$, and the fact that the ratio $E_{\text{fb}}^{11\text{hr}}/E_{\text{fb}}^{3\text{hr}}$ is consistent with unity, implies that no significant radiative cooling occurs between 3 and 11 hrs. In addition, the small intrinsic variance in E_γ/E_{fb} implies that if radiative losses are significant at earlier times, they are nearly uniform among all GRBs. These observations can be used to impose constraints on parameters of the fireball afterglow model. We demonstrate this by using the model of Berger et al. (2004) for radiative losses, in which the fireball energy drops with time as $E_{\text{fb}} \propto t^{-17\varepsilon/12}$ with $\varepsilon = \varepsilon_e/(1 + 1.05\varepsilon_e)$, as long as all electrons are in the fast cooling regime.

We first note that radiative losses cannot account for most of the scatter between fireball energy estimates at 3 and 11 hours, since the average and std

of $\log_{10}(E_{\text{fb},3\text{hr}}/E_{\text{fb},11\text{hr}})$ are 0.04 and 0.28 respectively, while radiative losses can only make this ratio larger than unity. Thus, we assume that only the average value is affected by radiative losses, i.e. on average no more than 20% of the fireball energy is radiated away between 3 and 11 hours. This implies either that ε_e is small, $\varepsilon_e < 0.15$, in which case radiative losses are small altogether, or that the stage at which all electrons are in the fast cooling regime ends at $t < 1$ hr. Using the estimate of Sari et al. (1998) for the duration of this fast cooling stage, t_{fast} , together with our estimate of $\varepsilon_e E_{\text{fb}}$, we find $n = 2 \times 10^{-2}(\varepsilon_B/0.1)^{-2}(\varepsilon_e/0.3)^{-1}(t_{\text{fast}}/1 \text{ hr})\text{cm}^{-3}$.

To conclude, the small difference ($< 20\%$) between the estimated fireball energies at 3 and 11 hours implies either $\varepsilon_e \lesssim 0.15$, or a circum burst density $n \lesssim 2 \times 10^{-2}(\varepsilon_B/0.1)^{-2}\text{cm}^{-3}$.

6.6. Energy injection

The tight correlation between $E_{\text{fb}}^{11\text{hr}}$ and $E_{\text{fb}}^{3\text{hr}}$, and the fact that the ratio $E_{\text{fb}}^{11\text{hr}}/E_{\text{fb}}^{3\text{hr}}$ is consistent with unity, implies that no significant injection of energy takes place during this time period. Can significant energy injection at yet earlier time be a common feature of GRBs?

Since some the afterglow lightcurves contain periods of shallower decay than expected by the basic fireball model, it has been suggested that additional energy might be injected to the fireball at times later than the initial burst. Based on their analysis of 9 lightcurves (out of a sample of 27) containing a phase of shallow decay, Nousek et al. (2006) suggest that energy injection might increase the fireball energy by a factor ≥ 4 on a timescale mostly up to ≈ 3 hours. In fact, their analysis (see their table 3) suggests that for these burst, the increase in fireball energy can even be much more significant, on the order of a factor of 10 on average (which, if taking into account radiative losses which occur simultaneously, could actually correspond to an even more significant energy injection). The variance in the factor by which the energy is increased is $\sim 2 - 6$. Similarly, Zhang et al. (2006) present lightcurves containing a shallow decay phase, and interpret them as indications for late time energy injection, but they do not quantify how representative of the "average GRB" these examples are. In a more recent work focusing on energy injection, Panaitescu & Vestrand (2012) find that among a sample of ≈ 100 swift afterglows featuring a break in the X-ray lightcurve, $\approx 30\%$ of the breaks can be explained as being jet breaks in an adiabatic model, while $\approx 60\%$ can be explained as jet breaks in a model including extended energy injection, thus suggesting that energy injection on a time scale of up to ≈ 1 day is a ubiquitous feature of GRB afterglows.

Our results, on the other hands, favor the possibility that late time energy injection is not a significant feature in GRBs. First, the relatively low intrinsic variance which we find in the ratio of the prompt energy and of the fireball energy, E_γ/E_{fb} , implies that any significant energy injection must be correlated with the prompt emission, i.e. it should increase the fireball energy by a constant factor for all bursts. While we cannot rule out such a possibility, it would mean that the already challenging high efficiency of conversion of fireball energy to γ -rays should be even higher, which would then be difficult to

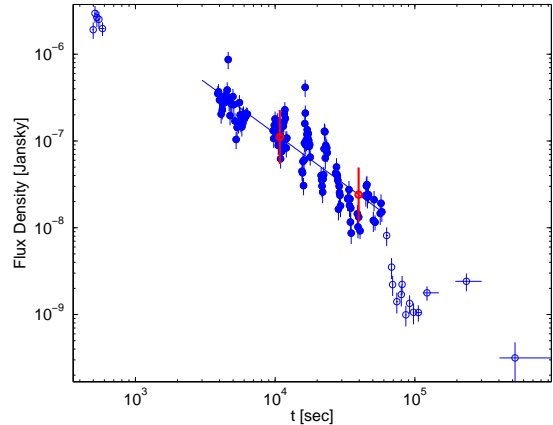


FIG. 9.— XRT data for GRB090516. The straight line is the power law fit, from which the flux values at 3 and 11 hours are inferred (red circles). The uncertainties in the inferred values (red lines) are based on the scatter of the X-ray data points. Open circles denote data points at times which are outside the time window which we use for determining the X-ray fluxes at 3 hr and 11 hr (3×10^3 s to 6×10^4 s for all bursts in the sample).

explain. Moreover, Panaitescu & Vestrand (2012) find a large scatter, of ≈ 2 orders of magnitude, in the factor by which energy injection increases the fireball energy, which is inconsistent with the small variance we find in E_γ/E_{fb} .

Second, the significant energy injection inferred by Panaitescu & Vestrand (2012) during the time period of 100 s to 100 kilo-s implies an average increase in energy by a factor of 1.5 – 3.5 between 3 hours and 11 hours, inconsistent with our finding that $E_{\text{fb}}^{11\text{hr}}$ and $E_{\text{fb}}^{3\text{hr}}$ are the same on average. Third, their analysis is based on detecting breaks in the lightcurves, which are interpreted as jet breaks which occur at $t < 11$ hr for 60% of the cases. This is inconsistent with our finding that jet breaks do not typically occur at such early time.

Finally, looking specifically at the bursts common to our sample and to the sample of Panaitescu & Vestrand (2012), and which feature a ≥ 3 hours break, which according to their analysis can only be explained with the presence of late time energy injection, we find the estimated fireball energies at 3 and 11 hours to be similar. As an example, the XRT data for GRB090516, for which Panaitescu & Vestrand (2012) find a break at 2×10^4 s, is shown in figure 9. In our analysis, the energy estimates at 3 and 11 hours are similar for this GRB, and the break, which is not analysed in the current work, appears to occur later.

6.7. Inverse-Compton losses

If the electrons responsible for the emission of X-rays via synchrotron emission lose a significant fraction of their energy by inverse-Compton (IC) emission at much higher frequencies, this would lead to a significant under estimate of the fireball energy E_{fb} . As was already noted by Berger et al. (2003), since the effect of IC losses depends strongly on n (and on ε_B), we would expect it to vary strongly between bursts, leading to a large scatter in E_γ/E_{fb} . The small scatter in E_γ/E_{fb} suggests that IC losses are not significant at 3 and 11 hrs. We do not carry a detailed analysis of the implications of this conclusion

to fireball model parameters, but note that IC emission may be suppressed by the Klein-Nishina effect for reasonable model parameters (e.g. Nakar, Ando & Sari 2009).

7. CONCLUSIONS

We have analyzed a sample of 91 *swift* Gamma-Ray Bursts (GRBs) with known redshifts, and found E_γ and E_{fb} to be tightly correlated and $E_{\text{fb}}^{3\text{hr}}$ and $E_{\text{fb}}^{11\text{hr}}$ to be tightly correlated (§ 5). The average(std) of $\eta_\gamma^{11\text{hr}} \equiv \log_{10}(E_\gamma/(3\varepsilon_e E_{\text{fb}}^{11\text{hr}}))$ are $-0.34(0.60)$, and the upper limit on the intrinsic spread of η_γ is approximately 0.5. If the uncertainties in the determinations of E_γ and E_{fb} are twice larger than we estimated, then the data imply no intrinsic variance in η_γ . The average(std) of $\log_{10}(E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{11\text{hr}})$ are $0.04(0.28)$. Given our estimated uncertainties in inferring E_{fb} , the data are consistent with no intrinsic variance in the ratio $E_{\text{fb}}^{3\text{hr}}/E_{\text{fb}}^{11\text{hr}}$.

The implications of these results were discussed in detail in § 6. The small variance of η_γ implies that burst-

to-burst variations in ε_e and in the efficiency of fireball energy conversion to γ -rays are small, and suggests that both are of order unity. It also implies that burst-to-burst variations in p are small. The small variance of η_γ and the similarity of $E_{\text{fb}}^{3\text{hr}}$ and $E_{\text{fb}}^{11\text{hr}}$ further imply that ε_e and p do not vary significantly with shock Lorentz factor, and that for most bursts the modification of fireball energy during the afterglow phase, by processes such as radiative losses or extended duration energy injection, are not significant (except possibly for a minority, $\sim 5\%$, of the bursts). Finally, our results imply that if fireballs are indeed jets, then the jet opening angle satisfies $\theta \geq 0.1$ for most cases.

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